Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 6, Issue 2; January-March, 2019, pp. 141-146

http://www.krishisanskriti.org/Publication.html

Finite Amplitude Analysis of Maxwell Fluid in Porous Medium in Presence of Soret and Dufour Effects under LTNE Model

R. Bansal[1], S. Kapoor[2] and S. Rawat[3]

[1]Ph.D Student, Department of Mathematics, IFTM University Moradabad
[2]Department of Education in Science and Mathematics, Regional Institute of Education (NCERT), Bhubneshwar, India
[3]Assistant Professor & Course Director (Mathematics Group), Department of General Studies, Jubail University College (Male Branch)
E-mail: [1]rg12794@gmail.com, [2]saurabh09.iitr@gmail.com, [3]sam.rawat@gmail.com

Abstract—*In this paper the finite amplitude analysis is done for the double diffusive free convection of Maxwell viscoelastic fluid in a porous medium in the presence of temperature gradient (Soret effects) and concentration gradient (Dufour effects) under LTNE model is investigated. The normal mode analysis is adopted for the linear stability analysis. The nonlinear analysis using a truncated representation of Fourier series considered only for two terms. The heat and mass transport phenomena is also depicted in this work. Graphical representation of physical parameter is also given in this paper.*

1. INTRODUCTION

Double diffusive convection in a horizontal layer of Maxwell viscoelastic fluid in a porous medium in the presence of temperature gradient (Soret effects) and concentration gradient (Dufour effects) is investigated. For the porous medium Darcy model is considered. A linear stability analysis based upon normal mode technique is used to study the onset of instabilities of the Maxwell viscolastic fluid layer confined between two free-free boundaries. Rayleigh number on the onset of stationary and oscillatory convection has been derived and graphs have been plotted to study the effects of the Dufour parameter, Soret parameter, Lewis number, and solutal Rayleigh number on stationary convection. The finite amplitude analysis is also done in this work for the flow stability

Non-Newtonian fluids have been a famous topic of research for their diverse use in many industrial processes, such as polymer solutions, blood, and heavy oils. These fluids have been modeled in a number of diverse manners with their constitutive equations varying greatly in complexity, among which the viscoelastic Maxwell fluid model has been studied widely[1–3]. The Maxwell fluid has achieved some successes in describing polymeric liquids, in which case it is more amenable to analysis and more important to experiments. The relaxation and retardation functions were determined for the four-parameter Maxwell model by Friedrich[4]. Song and Jiang[5] used the fractional calculus to analyze the experimental data of viscoelastic gum and obtained satisfactory results.

Qi and Xu[6–7] considered Stokes' first problem and some unsteady unidirectional flows for a viscoelastic fluid with the generalized Oldroyd-B model. Zheng et al.[8–9] considered some MHD flows of the generalized viscoelastic fluid. Shen et al.[10–11] studied the decay of vortex velocity and diffusion of temperature in a generalized second grade fluid and a Reyleigh-Stokes problem for a heated generalized second grade fluid with the fractional derivative. Recently, some new energy constitutive equation models have been proposed by Ezzat[12]

2. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of Maxwell viscoelastic fluid of thickness "d," confined between the planes z = 0 and z = d in a porous medium of porosity ε and medium permeability

k 1 and is acted upon by gravity $\mathbf{g}(0, 0, -g)$. This layer of fluid is heated and soluted in such a way that a constant temperature and concentration distribution is prescribed at the boundaries of the fluid layer. The temperature (T) and concentration (C) are taken to be T 0 and C

0 at z = 0 and T=1 and C be the difference in temperature and concentration across the boundaries.

Let $\mathbf{q}(u$, V, $w)$, p, ρ, T, C, α, α□, μ, κ, and κ□ be the Darcy velocity vector, hydrostatic pressure, density, temperature, solute concentration, coefficient of thermal expansion, an analogous solvent coefficient of expansion, viscosity, thermal diffusivity, and solute diffusivity of fluid, respectively.

2.1. Assumptions

The mathematical equations describing the physical model are based upon the following assumptions.

(i) Thermophysical properties expect for density in the buoyancy force (Boussinesq hypothesis) are constant.

(ii) Darcy's model with time derivative is employed for the momentum equation.

(iii) The porous medium is assumed to be isotropic and homogeneous.

(iv) No chemical reaction takes place in a layer of fluid.

(v)The fluid and solid matrix are in thermal equilibrium state.

(vi) Radiation heat transfer between the sides of the wall is negligible when compared with other modes of the heat transfer.

2.2. Governing Equations

According to the works of Bhatia and Steiner [14, 15], Sharma and Kumar [16], and Chand [19–21] the appropriate governing equations for Maxwell viscoelastic fluid in a porous medium are

$$\nabla . q = 0 \tag{1}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t}+\nabla p-\rho\, g\right)\right)+\frac{\mu}{K}q = 0 \ (2) \tag{2}$$

$$\varepsilon\,(\rho_0 c)_f\frac{\partial T_f}{\partial t}+(\rho_0 c)_f(q.\nabla)T_f = \varepsilon k_f\nabla^2 T_f + h(T_s - T_f) \tag{3}$$

$$(1-\varepsilon)(\rho_0 c)_s\frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s - h(T_s - T_f) \tag{4}$$

$$\varepsilon\frac{\partial C}{\partial t}+q.\nabla C = k'\nabla^2 C + D_{CT}\nabla^2 T, \tag{5}$$

$$\rho = \rho_0[1-\beta_T(T_f - T_l)+\beta_S(C - C_l) \tag{6}$$

where $q(u, v, w)$ is the Darcy velocity, p is the pressure, g is the acceleration due to gravity, μ is the viscosity, $\boldsymbol{\lambda}$ is the relaxation time, ρ is the density while K and $\boldsymbol{\varepsilon}$ are the permeability and porosity of the medium, T and C are the temperature and concentration, respectively

Where $\boldsymbol{D_{CT}}$ is Soret coefficients ; $\boldsymbol{\sigma} = (\boldsymbol{\rho c_p})_{\boldsymbol{m}}/(\boldsymbol{\rho c_p})_{\boldsymbol{f}}$ is the thermal capacity ,Where $(\boldsymbol{\rho c_p})_{\boldsymbol{f}}$is the volumetric heat capacity of the fluid and= $(\boldsymbol{\rho c_p})_{\boldsymbol{m}} = (\boldsymbol{1}-\boldsymbol{\varepsilon})(\boldsymbol{\rho c})_{\boldsymbol{s}} + \boldsymbol{\varepsilon}(\boldsymbol{\rho c})_{\boldsymbol{f}}$ is the volumetric heat capacity of the saturated medium as a whole, with the subscripts $\boldsymbol{f, s, and\ m}$ denoting the properties of the fluid, solid, and porous matrix, respectively. k is solutal diffusivity of the medium $\boldsymbol{D_{CT}}$ are Soret coefficients. β_T and β_S thermal and solute expansion coefficient in the medium.

We assume that temperature and concentration are constant at the boundaries of the fluid layer. Therefore, boundary condition are

$$w = 0, T = T_o\,, C = \ C_o \ at\ z = 0$$

$$w = 0, T = T_1\,, C = \ C_1 \ at\ z = d \tag{7}$$

2.3 ***Steady state and its solutions.*** The steady state is given by

$$u = v = w = 0, p = p(z), T = T_s(z), C = C_s\,(z)\ q_s = (0,0,0)\,, p = p_s(z), T = T_s(z), C = C_s\,(z), \rho = \rho_s(z), h = 0 \tag{8}$$

The basic state temperatures and concentration satisfy the equations

$$\frac{d^2T_{fb}}{dz^2} = 0\,, \frac{d^2T_{sb}}{dz^2} = 0\,, \frac{d^2C_b}{dz^2} = 0 \tag{9}$$

with boundary condition

$$T_{fb} = T_{sb} = T_l \text{ and } C_b = C_l\ at\ z = 0$$

$$T_{fb} = T_{sb} = T_u \text{ and } C_b = C_u\ at\ z = d \tag{10}$$

so that the conduction state solutions are given by

$$T_{fb} = T_{sb} = -\Delta T\left(\frac{z}{d}\right)+T_l, \tag{11}$$

$$C_b = C_l - \Delta C\left(\frac{z}{d}\right) \tag{12}$$

2.4 Perturbation Solution

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms.

$$q = 0 + q',\ T_f = T_{fb} + T_f',\quad T_s = T_{sb} + T_s',\ \ C = C_b + C', p = p_b + p', \rho = \rho_b + \rho', \tag{13}$$

where the prime denotes the perturbed quantities. Substituting (13) into (1-6) and neglecting higher order terms of the perturbed quantities, we obtain the equations governing the perturbations in the form

$$\nabla . q' = 0 \tag{14}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t}+\nabla p' + \rho_0(\beta_T T' - \beta_C C')\ g)\right)+\frac{\mu}{K}q' = 0 \ (15)$$

$$\varepsilon\,(\rho_0 c)_f\frac{\partial T_f'}{\partial t}+(\rho_0 c)_f(q'.\nabla)T_f' + (\rho_0 c)_f\, w'\left(\frac{d\,T_{fb}}{dz}\right) = \varepsilon k_f\nabla^2 T_f' + h(T_S' - T_f') \tag{16}$$

$$(1-\varepsilon)(\rho_0 c)_s\frac{\partial T_S'}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_S' - h(T_S' - T_f') \tag{17}$$

$$\varepsilon\frac{\partial C'}{\partial t}+(q'.\nabla)C' - w'\frac{\Delta C}{d} = k'\nabla^2 C' + D_{CT}\nabla^2 T', \tag{18}$$

By operating curl twice on Eq. (15), we eliminate p' from it and then render the resulting equation and other equations dimensionless using the transformations and non dimensional parameters

3. NORMAL MODES AND STABILITY ANALYSIS

Analyze the disturbances into the normal modes and assume that the perturbed quantities are of the form

$$\begin{pmatrix} w \\ T_f \\ T_S \\ C \end{pmatrix} = e^{\omega t} \begin{pmatrix} \psi \sin(ax) \\ \Theta \cos(ax) \\ \Phi \cos(ax) \\ \Sigma \cos(ax) \end{pmatrix} \sin(\pi z)$$

(19)

where k_x, k_y are wave numbers along x and y directions, respectively, and n is growth rate of disturbances. Using (19),

$$\begin{bmatrix} \delta^2(\frac{\omega}{Va}+1) & -aRa_T & 0 & aRa_S \\ -a & \omega+\delta^2+H & -H & 0 \\ 0 & -\gamma H & \alpha\omega+\delta^2+\gamma H & 0 \\ -a & 0 & S_r\delta^2 & \frac{\delta^2}{Le}+\omega \end{bmatrix} \begin{bmatrix} \psi \\ \Theta \\ \Phi \\ \Sigma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(20)

$$\delta^2 = \pi^2 + a^2$$

Substituting solution (13) in (11), integrating each equation from z = 0 to z = 1 by parts, we obtain the following matrix equation as

The nontrivial solution of the above matrix requires that

$$Ra_T = \frac{\delta^2(\frac{\omega}{Va}+1)[(\alpha\omega+\delta^2+\gamma H)(-\alpha\omega+\delta^2+\gamma H)-H^2\gamma]}{a^2(\alpha\omega+\delta^2+\gamma H)} - \frac{a^2[(\alpha\omega+\delta^2+\gamma H)^2-H^2\gamma]+[a\gamma HS_r\delta^2]}{a^2(\alpha\omega+\delta^2+\gamma H)(\omega+\frac{\delta^2}{Le})} Ra_S \quad (21)$$

It is clear from (21) that stationary Rayleigh number Ra is a function of dimensionless wave number a, Soret parameter S r, Lewis number Le and solutal Rayleigh number Rs, and independent of stress relaxation parameter. Thus for stationary convection the Maxwell viscoelastic fluid behaves like an ordinary Newtonian fluid.

4. FINITE AMPLITUDE STEADY CONVECTION

In this section we consider the nonlinear analysis using a truncated representation of Fourier series considering only two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigen functions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. To obtain this additional information, we perform the nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward towards understanding full nonlinear problem. There is a large body of literature available on the finite amplitude thermal convection in porous medium with local thermal equilibrium condition for both single and two component systems .The method proposed by these authors has been adopted here in this paper to study the effect of local thermal non-equilibrium on double diffusive convection in a porous layer. A minimal double Fourier series which describes the finite amplitude steady-state convection is given by

$$\psi = A\sin(ax)\sin(\pi z)$$

$$T_f = B_1\cos(ax)\sin(\pi z) + B_2\sin(2\pi z)$$

$$T_S = B_3\cos(ax)\sin(\pi z) + B_4\sin(2\pi z)$$

$$C = B_5\cos(ax)\sin(\pi z) + B_6\sin(2\pi z) \quad (22)$$

where the steady-state amplitudes A, Bi 's are constants and are to be determined from the dynamics of the system. Substituting Eqs. (22) into the steady part of coupled nonlinear system of partial differential equations (20) and equating the coefficients of like terms we obtain the following nonlinear system equations

$$(\pi^2+a^2)A + aRa_TB_1 - aRa_SB_5 = 0 \ (23)$$

$$aA + [(\pi^2+a^2)+H]B_1 - HB_3 + \pi aAB_2 = 0 \quad (24)$$

$$2[4\pi^2+H]B_2 - 2HB_4 - \pi aAB_1 = 0 \quad (25)$$

$$\gamma HB_1 - [(\pi^2+a^2)+\gamma H]B_3 = 0 \ (31)$$

$$\gamma HB_2 - [4\pi^2+\gamma H]B_4 = 0 \quad (26)$$

$$aA + \frac{1}{Le}(\pi^2+a^2)B_5 + \pi aAB_6 = 0 \quad (27)$$

$$\frac{8\pi^2}{Le}B_6 - \pi aAB_5 = 0 \quad (28)$$

The steady state solutions are useful because they predict that a finite amplitude solution to the system is possible for subcritical values of the Rayleigh number and that the minimum values of Ra_T for which a steady solution is possible lies below the critical values for instability to either a marginal state or an overstable infinitesimal perturbation. Elimination of all amplitudes, except A, yields $A\Big\{(\pi^2+a^2) - a^2Ra_T\left[\frac{(\pi^2+a^2)\{\pi^2+a^2+H(1+\gamma)\}}{(\pi^2+a^2+\gamma H)} + \frac{a^2(4\pi^2+\gamma H)}{4\pi^2+H(1+\gamma)}\left(\frac{A^2}{8}\right)\right]^{-1} + a^2Ra_S\left[\frac{(\pi^2+a^2)}{Le} + a^2Le\left(\frac{A^2}{8}\right)\right]^{-1}\Big\} = 0$ (29) The solution $A = 0$ corresponds to pure conduction, which we know to be a possible solution though it is unstable when Ra_T is sufficiently large. The remaining solutions are given by

$$\frac{A^2}{8} = \frac{1}{2x_1}\left[-x_2 + \sqrt{x_2^2 - 4x_1x_3}\,\right] \quad (30)$$

When we let the radical in the above equation to vanish, we obtain an expression for finite amplitude Rayleigh number Ra_T^F, which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra_T^F = \frac{1}{2y_1}\left[-y_2 + \sqrt{y_2^2 - 4y_1y_3}\,\right] \quad (31)$$

5. RESULT AND DISCUSSION

An analytical study of linear and nonlinear double diffusive convection in a horizontal fluid-saturated sparsely packed porous layer is carried out by considering a thermal non-equilibrium model. The onset criterion for both marginal and oscillatory convection is derived using the linear theory. The expression for finite amplitude Rayleigh number is derived analytically using minimal representation of the Fourier series. The effect of solute diffusion, and thermal non-equilibrium on the stability of the system is investigated. It is found that in most of the situations the instability sets in via finite amplitude motions, prior to the stationary or oscillatory convection.

The variation of the critical Rayleigh number, wavenumber and frequency of oscillations with *H* for different values of Vadasz number is unveiled in Figs. 1a–c. We observe from Fig. 1 a that the oscillatory critical Rayleigh number decreases with increasing Va. The effect of Vadasz number is therefore, to advance the onset of double diffusive convection, in oscillatory mode. In Fig. 1b we display the effect of Vadasz number on the oscillatory critical wavenumber. This figure indicates that the critical wavenumber *a* Osc c for oscillatory mode increases with Va. It is clear from Fig. 1c that the critical frequency increases with increasing Va.

Figure 2 a–c displays the effect of the Darcy number on the critical Rayleigh number, wave number and the frequency of the oscillations. Figure 2a indicates the effect of Darcy number *Da* on the critical Rayleigh number. We find from this figure that when *Da* is very small, i.e. in case of a densely packed porous medium, the finite amplitude motions occur prior to the oscillatory motions for small to moderate values of the interface heat transfer coefficient *H*, and for large *H*, convection sets in first as oscillatory mode. The fairly large *Da* (≥0.1) dampens the finite amplitude motions and the overstable mode becomes the most dangerous mode in such case. The critical Rayleigh number for stationary, oscillatory and finite amplitude convection is found to decrease with decreasing *Da*. The size of *Da* is related to viscous effects at the boundaries, and reduction in *Da* decreases this effect, which allows the fluid to move more easily, thereby decreasing the critical Rayleigh number. Figure 2b indicates the variation of critical wavenumber with *H* for different values of *Da*. We observe from this figure that for intermediate values of *H* the critical wavenumber for each of the stationary, oscillatory and finite amplitude modes attains the maximum value and decreases with increasing *Da*. However, the small and large *H* has no influence on the critical wavenumber. Figure 2c show the effect of Darcy number on the amplitude of the oscillations. We find that the effect of increasing *Da* is to decrease the amplitudes. In the study of double diffusive convection the determination of heat and mass transport across the layer plays a vital role. Here, the onset of convection as the Rayleigh number is increased is more rapidly detected by its effect on the heat and mass transfer.

6. CONCLUSIONS

The linear and nonlinear double diffusive convection in a horizontal fluid-saturated sparsely packed porous layer is investigated analytically when the fluid and solid phases are not in LTE. We have analyzed in detail the combined effects of boundary and LTNE on the onset of double-diffusive convection in a porous layer. In case of linear theory the thresholds of both stationary and oscillatory convection are derived as the functions of solute Rayleigh number, inter-phase heat transfer coefficient, Lewis number, porosity modified conductivity ratio, Vadasz number, diffusivity ratio and Darcy number. The nonlinear theory predicts the occurrence of finite amplitude motions. We found that there is a competition between the processes of thermal and solute diffusion that causes the convective instability to set in as oscillatory and finite amplitude mode rather than stationary. It is found that for both large and small inter-phase heat transfer coefficient the system behaves like a LTE model while the intermediate values have strong influence on each of stationary, oscillatory and finite amplitude modes. The presence of a stabilizing gradient of solute will inhibit the onset of double diffusive convection. The magnitude of *Da* is related to the importance of viscous effects at the boundaries, and reduction in *Da* decreases this effect, thereby decreasing the critical Rayleigh number. The effect of porosity modified conductivity ratio, Vadasz number, is to enhance the instability of system. Each of the parameters *Ra*S*, H, γ, Le* and *Da* increases the values of *Nu* and *Sh.*

REFERENCE

[1] Fetecau, C., Athar, M., and Fetecau, C. Unsteady flow of a generalized Maxwell fluid with fractional derivative due to a constantly accelerating plate. *Computers and Mathematics with Applications*, **57**, 596–603 (2009)

[2] Hayat, T. and Sajid, M. Homotopy analysis of MHD boundary layer flow of an upper-convected Maxwell fluid. *International Journal of Engineering Science*, **45**, 393–401 (2007)

[3] Tan, W. C. and Masuoka, T. Stability analysis of a Maxwell fluid in a porous medium heated from below. *Physics Letters A*, **360**, 454–460 (2007)

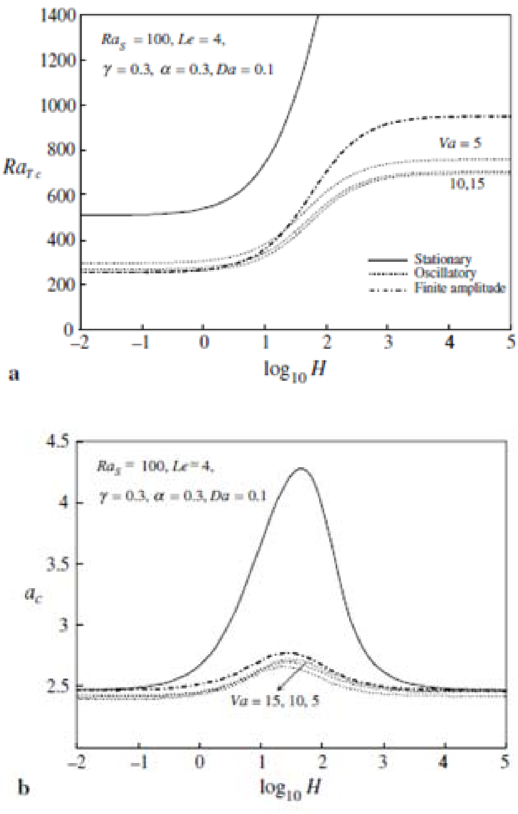
[4] Podlubny, I. *Fractional Differential Equations*, Academic Press, New York (1999) [5] Guo, B. L., Pu, X. K., and Huang, F. H. *Fractional Partial Differential Equations and Their Numerical Solutions*, Science Press, Beijing (2011)

[6] Bagley, R. L. and Torvik, P. J. A theoretical basis for the application of fractional calculus to viscoelasticity. *Journal of Rheology*, **27**, 201–210 (1983)

[7] Friedrich, C. Relaxation and retardation functions of the Maxwell model with fractional derivatives. *Rheologica Acta*, **30**, 151–158 (1991)

[8] Song, D. Y. and Jiang, T. Q. Study on the constitutive equation with fractional derivative for the viscoelastic fluids modified Jeffreys model and its application. *Rheologica Acta*, **27**, 512–517 (1998)

[9] Qi, H. T. and Xu, M. Y. Stokes' first problem for a viscoelastic fluid with the generalized Oldroyd- B model. *Acta Mechanica Sinica*, **23**, 463–469 (2007)
[10] Qi, H. T. and Xu, M. Y. Some unsteady unidirectional flows of a generalized Oldroyd-B fluid with fractional derivative. *Applied Mathematical Modeling*, **33**, 4184–4191 (2009)
[11] Zheng, L. C., Liu, Y. Q., and Zhang, X. X. A new model for plastic-viscoelastic magnetohydrodynamic (MHD) flow with radiation thermal transfer. *International Journal of Nonlinear Sciences and Numerical Simulation*, **14**, 435–441 (2013)
[12] Zheng, L. C., Liu, Y. Q., and Zhang, X. X. Exact solutions for MHD flow of generalized Oldroyd-B fluid due to an infinite accelerating plate. *Mathematical and Computer Modelling*, **54**, 780–788 (2011)
[13] Cortell, R. Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. *Physics Letters A*, **371**, 631–636 (2008)
[14] Ezzat, M., El-Bary, A. A., and Ezzat, S. Combined heat and mass transfer for unsteady MHD flow of perfect conducting micropolar fluid with thermal relaxation. *Energy Conversion and Management*, **52**, 934–945 (2011)



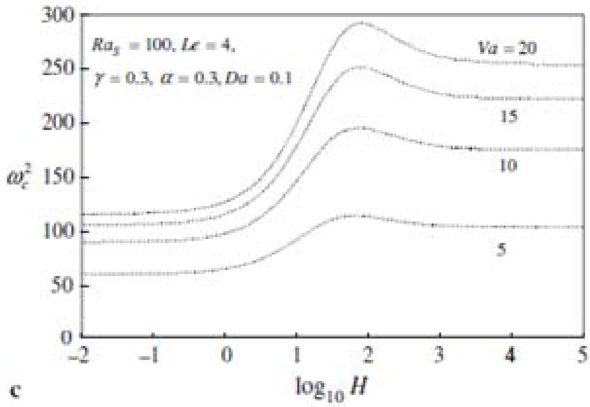
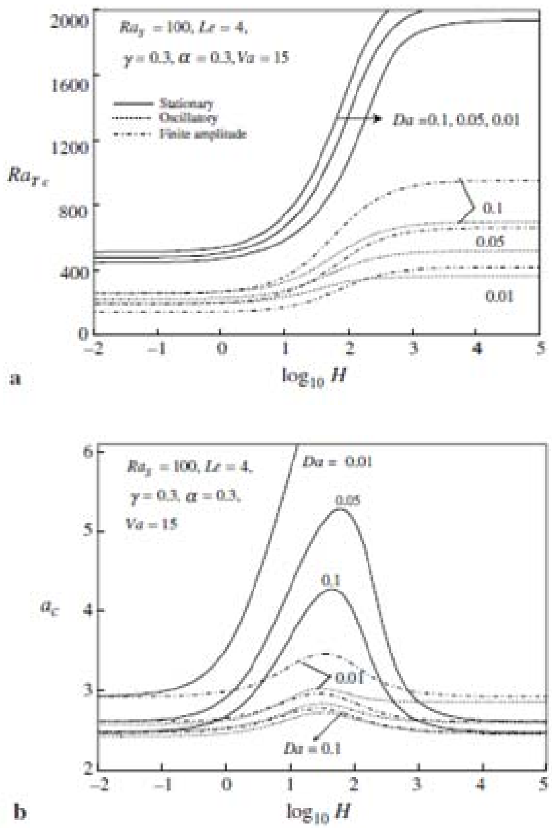


Figure 1 The variation of the critical Rayleigh number, wavenumber and frequency of oscillations with H for different values of Vadasz number



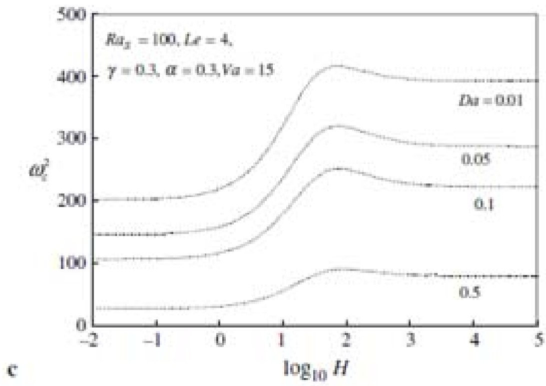


Figure 2 Variation of critical values of Rayleigh number, wave number and the frequency for different values of darcy number.